

# The Effect of Wage Risk and Cross-Sectional Aggregation on the Labor Supply Response in Macroeconomic Models<sup>1</sup>

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## Abstract

We evaluate the labor supply response in a stochastic overlapping generations model with incomplete markets and a balanced social security system with payroll taxes and benefit claims that depend on the actual earnings history. The model is calibrated with PSID data to fit the main features of the US economy including the fraction of borrowing constrained households. First, we calculate the labor supply response to anticipated changes in wages holding the marginal utility of wealth constant (Frisch elasticity) using methods from the empirical literature and compare them with the elasticity implied by the individual's theoretical problem. In line with previous studies, we find that they differ by a factor of 1.5 to 6. Higher order approximation terms that result from mapping the individual problem to the empirical specification account for between 71% and 93% of that difference, while borrowing constraints account for the rest, a result in contrast to Domeij and Floden (2006). Second, identified within the higher order approximation terms, we calculate the labor supply response to unanticipated changes in wages (the effect of wage shocks and wage uncertainty) finding small responses due to large and age-decreasing wealth effects. Omitting measures of wage uncertainty biases the estimates of the Frisch elasticity toward zero. We also calculate the macroeconomic (cross sectional aggregation) Frisch elasticity at the extensive and intensive margins. The extensive margin macro Frisch elasticity is 0.17. The intensive margin macro Frisch elasticity is 1.4, but suffers from bias due to cross sectional aggregation (determined by Thail's entropy index). Once this bias is corrected, the estimate drops to 0.7. Adding both margins, the macro Frisch elasticity is 0.87, lower than the value suggested by other macroeconomic studies.

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Keywords:

labor supply, intertemporal substitution, Frisch elasticity, stochastic GE models

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# 1 Introduction

The validity of dynamic macroeconomic models for analyzing policy experiments depends on key behavioral parameters that determine the agents' responses to changes in policy variables. The intertemporal elasticity of labor supply, which determines the labor supply response, is among the most important of those parameters.

In this paper, we analyze the empirical validity of the labor supply response in a stochastic overlapping generations (OLG) model with incomplete markets, a balanced social security system with payroll taxes and benefit claims that depend on the actual earnings history. The model allows for the labor supply responses at the intensive margin (by changing the number of hours worked) and extensive margin (by working zero hours at any point in the life cycle). The model is calibrated with PSID data to fit the main features of the US economy including the fraction of households that have borrowing constraints.<sup>2</sup> We use simulated microeconomic data from the model to analyze the labor supply response to anticipated changes in wages, holding constant the marginal utility of wealth (Frisch elasticity), and to unanticipated changes in wages.<sup>3</sup> We also analyze the estimates of the Frisch elasticity using macroeconomic data from the model (cross-sectional aggregation).<sup>4</sup>

This paper makes three main contributions. First, it reveals that higher order approximation terms that reflect the uncertainty of the economy, rather than borrowing constraints, are the main factor responsible for the difference between the empirical mi-

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<sup>2</sup>The model is similar to those of Nishiyama (2004), Nishiyama and Smetters (2005) and CBO (2008).

<sup>3</sup>Economists usually consider four basic elasticities of labor supply (Blundell and MaCurdy [2008]): compensated and uncompensated elasticity of labor supply, Frisch elasticity of labor supply, and a life cycle elasticity that considers unanticipated (parametric) changes in wage changes. Because the compensated and uncompensated elasticities of labor supply usually are derived and estimated from static models, they are unsuitable for application in dynamic macroeconomic models. The Frisch elasticity of labor supply is defined as the percentage change in labor supply resulting from a one percent increase in the expected wage rate, holding the marginal utility of wealth constant. The Frisch elasticity measures the response of wages to anticipated wage changes, as opposed to the other life cycle elasticity, which measures the response of work hours to unanticipated wage changes or shifts in the wage profile.

<sup>4</sup>In the text we use the term Frisch elasticity to refer to the calculation of the intertemporal elasticity estimated with microeconomic data. When we consider macroeconomic data (cross-sectional aggregation) we refer explicitly to the extensive (or intensive) macro Frisch elasticity.

macroeconomic estimates of the Frisch elasticity and the values implied by the individual's theoretical maximization problem, a result that is the reverse of Domeij and Floden (2006). That result is due to the use of an OLG model that allows for labor supply responses at the extensive and intensive margin, and that closely replicates the main characteristics of the US economy, including the Social Security system and the prevalence of borrowing-constrained agents present in the economy (all of which affect the amount of insurance households can get in addition to self-insurance through savings). The second contribution is to characterize and quantify the importance of wage risk and wage uncertainty within the higher order approximation terms to analyze the labor supply response in this class of models. The third contribution is to generate macroeconomic estimates (via cross sectional aggregation) of the Frisch elasticity at the extensive margin that are in line with microeconomic studies that consider participation constraints, and that are well below the values that other macroeconomic models generate. In the same line, we quantify the bias induced by estimating the aggregate Frisch elasticity at the intensive margin using macroeconomic (cross-sectional aggregation) instead of microeconomic (individual-level) data.

We use several methods from the empirical literature to calculate the Frisch elasticity and compare them with the elasticity implied by the individual's theoretical maximization problem. The model's Frisch elasticity estimate is between 0.19 and 0.77 depending on the empirical method used,<sup>5</sup> a range slightly higher than the empirical estimates in the earlier literature but lower than some more recent estimates.<sup>6</sup> The theoretical elas-

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<sup>5</sup>We estimate the leisure elasticity instead of the labor supply elasticity to avoid further specification biases in our calculations, but they are directly comparable as will be clear in section 3.

<sup>6</sup>Empirical measures of the intertemporal Frisch elasticity of labor supply present a huge variation. Estimates range from close to zero to about 3, although most have values of less than 1. MaCurdy's (1981) estimates of the Frisch elasticity of labor supply are between 0.1 and 0.45. Altonji's (1986) estimates are between 0 and 0.35. Pencavel (1986) surveys this early literature and presents a mean value of 0.2 and a range of 0 to 0.45. Angrist (1991) estimated a Frisch labor supply elasticities of 0.6 to 0.8. French (2004) finds a Frisch elasticity close to zero with a standard error of 0.25. Most recent studies tend to find higher values than was the case earlier. Estimates generated by Ziliak and Kneisner (2005) are around 0.5. Pistaferri (2003) estimates a Frisch elasticity of 0.75 Fehr and Götte (2007) estimate an intertemporal elasticity of substitution of about 1.1, although they use high-frequency data and estimate an elasticity that does not hold the marginal utility of wealth constant, reason why their results are not relevant for the model used in this paper or for the calculation of the Frisch elasticity. Using a structural approach, Gourinchas and Parker (2002) present estimates that range from 0.7 to almost 2, Imai and Keane (2004) report that accounting for human capital accumulation can lead to

ticity is 1.14, between 1.4 to 6 times bigger than the empirical estimates, a result in line with previous work. Higher order approximation terms resulting from the mapping of the individual maximization problem to the empirical specifications account for between 71% and 93% of that difference, while borrowing constraints account for between 7% and 29% of the bias, which is in contrast to the result of Domeij and Floden (2006). We also find that lagged wages induce lower biases than life cycle instruments in the estimated Frisch elasticity. This happens because lagged wage instruments capture part of the uncertainty included in the higher order terms through wage risk.

Domeij and Floden (2006) argue that when the econometrician ignores borrowing constraints, the estimated intertemporal Frisch elasticities have a downward bias. They also identify an additional downward bias in many studies from the approximation errors of log-linearizing the Euler equation, but find that in their model higher order approximation errors are less important. We find the opposite: higher order approximation errors are much more important than borrowing constraint. This difference comes mainly from the type of model used. In our model, accidental bequests and the social security system are additional and important forms of consumption insurance through the lifecycle (OLG framework), while in Domeij and Floden the only insurance available is self-insurance in an infinite horizon framework. Borrowing constraints are important because without them households respond to a bad wage shock by reducing labor and increasing borrowing so as to smooth consumption over time. With borrowing constraints, however, households can't completely smooth consumption, so they don't reduce labor supply as much as they otherwise would. That dampens the labor supply response in the model. With the additional sources of insurance mentioned above, borrowing constraints do not dampen labor supply as much as when savings is the only source of insurance.

Many macroeconomic models focus on the Frisch elasticity values because their goal is to analyze real business cycles. Equally important but less studied is the life cycle elasticity that considers labor supply responses to unanticipated changes in wages.<sup>7</sup> This huge Frisch elasticities of labor supply (more than 3) and Laitner and Silverman (2005) present estimates that are around 0.9.

<sup>7</sup>Few studies, among them MaCurdy (1981) and Pistaferri (2003), estimate the intertemporal elasticity of labor supply in response to unanticipated wage changes. MaCurdy estimates a male labor

elasticity is also very important for the study and the analysis of tax and benefit reforms, which almost always cause unanticipated shifts in wages today and in the future (see Blundell and MaCurdy 1999 and 2008). Of course, the relative importance of the two elasticities depends on the type of tax policy implemented and on the mix of wealth and intertemporal substitution effects it triggers.

We also calculate the labor supply response to unanticipated changes in wages (i.e the effect of wage shocks and wage uncertainty) and find small responses due to large and age-decreasing wealth effects. Those shocks are part of the higher-order terms that arise from the mapping of the labor supply response in the individual maximization problem to the empirical specification. Having all the variables required and no measurement error, we calculate the omitted variable bias of not controlling for the level (shocks) and variance (uncertainty) of the unexpected changes in wages. Omitting wage uncertainty biases downwards the estimates of the Frisch elasticity.

Our estimates of the labor supply response to unanticipated wage changes are small because wealth effects in the model largely offset the intertemporal substitution effect. That result comes from important wealth effects in our OLG model. Unexpected wage changes shift the wage profile, generating a labor supply response from a wealth effect and from an intertemporal substitution (Frisch) effect. A wage increase reduces labor supply through a wealth effect, but the same wage increase generates an intertemporal substitution effect that tends to increase the labor supply. Depending on which effect is bigger, the labor supply response to unanticipated wage changes may be negative or positive. In our model, wealth effects have almost the same magnitude as intertemporal substitution effects.

Finally, we calculate the macro Frisch elasticity at the extensive margin (working or not) and at the intensive margin (number of hours worked) using cross-sectional aggregation of the participation decisions, hours and the wage rate. For the extensive margin, we obtain an estimate of 0.17, which is close to the microeconomic evidence on extensive margin responses (Chetty et al (2010) review this literature and find that the supply response to unanticipated wage changes of 0.08 in the median range; Pistaferri's estimate is 0.5.

average is of 0.28) For the intensive margin, we obtain a value of 1.4, 20% higher than the theoretical values and up to 7 times bigger than the empirical micro estimates. We show that this result is due to bias induced by the cross sectional aggregation. As in the consumption case (Attanasio and Weber 1993), this aggregation bias is determined by Thail's entropy index. Once this bias is corrected, the estimate drops to 0.7, half the original value. Chetty et al (2010) use the studies by Cho and Cooley (1994), King and Rebelo (1999), Smets and Wouters (2007) and Hall (2009) to arrive to a mean estimate of the macro Frisch elasticity of 2.61 (which combines labor supply response in both margins). Our results imply a total response of at most 0.87 (combining both the extensive and the intensive macro Frisch elasticity), which is in line with the evidence presented by Chetty et al (2010) for studies based in microeconomic data.

Our paper is related to Chang and Kim (2006), who use an infinite horizon model with incomplete markets and assume that consumption and leisure are separable in the utility function. In contrast, we use an OLG model and a CRRA utility function non-separable between leisure and consumption. Chang and Kim's utility function implies a direct mapping between the intertemporal substitution elasticity of leisure and the Frisch elasticity; the utility in our model does not. Another difference is that we can compare our results directly with previous results from the empirical literature because we apply standard econometric techniques used in previous microeconomic studies for comparison. Finally, Chang and Kim do not estimate the sources of bias present in the empirical studies nor the labor supply response to unexpected changes in wages.

This paper also is similar to the work of Domeij and Floden (2006) in that it considers the effects of uncertainty and borrowing constraints on the estimates of the Frisch elasticity of labor supply. It differs from Domeij and Floden (2006) in three major ways: First, we find that the difference between the theoretical Frisch elasticity and the empirical estimates comes from higher order approximation terms rather than from borrowing constraints. This result stems from the use of a life cycle model (instead of a model with infinitely lived agents) that includes accidental bequests and a social security system, providing extra sources of insurance through the lifecycle to consumers in addition to

self-insurance through saving. Second, we identify within the higher order terms the effects of wage shocks and wage uncertainty in the estimates of the Frisch elasticity using Pistaferri’s (2003) methodology. And finally, we calculate the macro Frisch elasticity at the intensive and extensive margins, identify the bias of the cross sectional aggregation of the labor supply responses at the intensive margin.

In the next section we describe the model and the calibration to fit the main features of the US economy. In section 3, we discuss methodological and empirical issues regarding the measurement of the Frisch elasticity at the individual level, present our estimates of various measures of intertemporal labor supply elasticities, quantify the effect of borrowing constraints and higher order approximation terms, and present the main source of bias from the empirical estimates. In section 4 we estimate the macro Frisch elasticity at the intensive and extensive margins and discuss the bias induced by using aggregate data instead of individual-level data to estimate the Frisch elasticity at the intensive margin. We present our conclusions in section 5.

## 2 The Model Economy

The model used in this paper is an overlapping generations model with heterogeneous agents, uninsurable idiosyncratic working ability shocks, mortality shocks, and a balanced social security system with payroll taxes and benefit claims that depend on the actual earnings history. The model is very similar to the model presented in Nishiyama (2004) and Nishiyama and Smetters (2005). We present the model in considerable detail, following closely the description in Nishiyama (2004), because many model characteristics (and not just the utility function) determine the labor supply response.

**Households.** Every period a large number of young new households enter the economy. A household’s state in period  $t$  is determined by  $\mathbf{s}_i = (i, e_i, a_i, b_i)$ , where  $i \in I = \{20, \dots, 120\}$  is the household’s age,  $e_i$  is its working ability (measured by its hourly wage),  $a_i$  is its beginning-of-period asset holding, and  $b_i$  is its average historical earnings.

A household of age  $i$  receives idiosyncratic working ability shock,  $e_i$ , at the beginning of each year  $t$ , and chooses its optimal consumption  $c_i$ , working hours  $h_i$ , and end-of-period wealth holding  $a_{i+1}$ . At the end of each year, a fraction  $1 - \phi_{i,t}$  of households die, and the survival rate at the end of age 120,  $\phi_{120,t}$ , is assumed to be zero.

Households take aggregate variables in the economy as given. The aggregate state of the economy is determined by  $\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{S,t}, W_{G,t})$ , where  $x_t(\mathbf{s}_i)$  is the measure of households,  $W_{S,t}$  is the beginning-of-period Social Security trust funds, and  $W_{G,t}$  is the rest of the government net wealth. The government policy is determined by  $\Psi_t = \{W_{S,t+1}, W_{G,t+1}, \tau_{PO,s}(\cdot), \tau_{PH,s}(\cdot), tr_{SS,s}(\cdot), \tau_{I,s}(\cdot), C_{G,s}\}_{s=t}^{\infty}$ , where  $\tau_{PO,s}(\cdot)$  is the Social Security payroll tax function,  $\tau_{PH,s}(\cdot)$  is the Medicare Hospital Insurance (HI) payroll tax function,  $tr_{SS,s}(\cdot)$  is the Social Security benefit function,  $\tau_{I,s}(\cdot)$  is a progressive income tax function, and  $C_{G,s}$  is government's consumption. Households also take the population projection  $\Phi_t$  as given.

The household's problem is:

$$v(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t) = \max_{c_i, h_i, a_{i+1}} u_i(c_i, h_i) + \beta \phi_{i,t} E \pi_{i,i+1}(e_{i+1}|e_i) [v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \Psi_{t+1}, \Phi_{t+1}) | e_i] \quad (1)$$

subject to:

$$\begin{aligned} a_{i+1} &= \frac{1}{1 + \xi} \{ \bar{w}_t e_i h_i + (1 + r_t) a_i - \tau_{I,t}(\bar{w}_t e_i h_i, r_t a_i, tr_{SS,t}(i, b_i)) \} \\ &\quad - \tau_{PO,t}(\bar{w}_t e_i h_i) - \tau_{PH,t}(\bar{w}_t e_i h_i) + tr_{SS,t}(i, b_i) - c_i \geq a_{i+1,t}^{\min}(\mathbf{s}_i), \\ a_{20} &= 0, \quad a_{i \in \{65, \dots, 121\}} \geq 0, \quad h^{max} \geq h_i \geq 0 \end{aligned} \quad (2)$$

where  $u_i(\cdot)$  is a period utility function of an age  $i$  household,  $\beta$  is the time-preference factor,  $\phi_{i,t}$  is the survival rate at the end of age  $i$ ,  $\bar{w}_t$  is the wage rate,  $\pi_{i,i+1}(e_{i+1}|e_i)$  is the conditional probability for the age  $i + 1$  working ability being  $e_{i+1}$  when the age  $i$  working ability is  $e_i$ , and  $r_t$  is the real rate of return to capital. Individual variables, except for working hours, are normalized by the steady-state per capita growth rate  $\xi$ .  $a_{i+1,t}^{\min}(\mathbf{s}_i)$  is the minimum sustenance level of end-of-period wealth. We calibrate this

level to match the percentage of households that are borrowing constrained. Under the no-borrowing-constrains economy, this parameter is the minimum asset level that allows a consumer to hold his consumption plan even if the household receives the worst possible working ability shocks for the rest of life.

The average historical earnings  $b_i$  follows

$$b_{i+1} = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24} \left\{ (i-25)b_i \frac{\bar{w}_t}{\bar{w}_{t-1}} + \min(\bar{w}_t e_i h_i / 2, \bar{w} e h_t^{\max}) \right\} & \text{if } 25 \leq i \leq 59 \\ (1 + \mu)^{-1} b_i & \text{if } i \geq 60, \end{cases} \quad (3)$$

where  $\bar{w} e h_t^{\max}$  is the Social Security payroll tax cap.

The measure of households is denoted as  $x_t(\mathbf{s}_i)$  and is adjusted by the long-run population growth rate  $\nu$ . The corresponding cumulative measure is denoted  $X_t(\mathbf{s}_i)$ . A household of age 20 is assumed to have no initial wealth and working history. So,  $\int_E dX_t(20, e_{20}, 0, 0)$  is the population of age 20 households in year  $t$ . Let  $\mathbf{1}_{[a=y]}$  be an indicator function, valued 1 if  $a = y$  and 0 if  $a \neq y$ . The law of motion of the measure of households is, for  $i = 20, \dots, 120$ ,

$$x_{t+1}(\mathbf{s}_{i+1}) = \frac{\phi_{i,t}}{1 + \nu} \int_{E \times A \times B} \mathbf{1}_{[a_{i+1}=a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t) + q_t]} \times \mathbf{1}_{[b_{i+1}=b_{i+1}(\bar{w}_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t), b_i)]} \pi_{i,i+1}(e_{i+1}|e_i) dX_t(\mathbf{s}_i). \quad (4)$$

A working age household is assumed to receive accidental bequests  $q_t$  with constant probability  $\eta$ , where  $q_t$  is the average wealth left by deceased households and  $\eta$  is the ratio of deceased household to the surviving working-age households. That is,

$$q_t = \frac{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}_i)}{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} dX_t(\mathbf{s}_i)}, \quad (5)$$

$$\eta = \frac{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} dX_t(\mathbf{s}_i)}{\sum_{i=20}^{64} \phi_i \int_{E \times A \times B} dX_t(\mathbf{s}_i)}. \quad (6)$$

**Aggregate Variables.** National wealth  $W_t$  is the sum of total private wealth, the Social Security trust funds  $W_{S,t}$ , and the rest of the government net wealth  $W_{G,t}$ . Total labor supply  $L_t$  is measured in efficiency units. Then,

$$W_t = \sum_{i=20}^{120} \int_{E \times A \times B} a_i dX_t(\mathbf{s}_i) + W_{S,t} + W_{G,t}, \quad (7)$$

$$L_t = \sum_{i=20}^{120} \int_{E \times A \times B} e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t) dX_t(\mathbf{s}_i). \quad (8)$$

Capital stock  $K_t$  is equal to national wealth, that is,  $K_t = W_t$ , and gross national product  $Y_t$  is determined by a constant-returns-to-scale production function, such that prices in the economy are determined by the profit-maximizing condition of the representative firm:

$$F_K(K_t, L_t) = r_t + \delta, \quad (9)$$

$$F_L(K_t, L_t) = \bar{w}_t, \quad (10)$$

where  $\delta$  is the depreciation rate of capital.

Government tax revenue consists of income tax,  $T_{I,t}$ , Social Security payroll tax,  $T_{PO,t}$ , and HI payroll tax,  $T_{PH,t}$ . Those revenues are calculated as

$$T_{I,t} = \sum_{i=20}^{120} \int_{E \times A \times B} \tau_{I,t} (\bar{w}_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t), r_t a_i, tr_{SS,t}(i, b_i)) dX_t(\mathbf{s}_i), \quad (11)$$

$$T_{PO,t} = \sum_{i=20}^{120} \int_{E \times A \times B} \tau_{PO,t} (\bar{w}_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t)) dX_t(\mathbf{s}_i), \quad (12)$$

$$T_{PH,t} = \sum_{i=20}^{120} \int_{E \times A \times B} \tau_{PH,t} (\bar{w}_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t, \Phi_t)) dX_t(\mathbf{s}_i). \quad (13)$$

Social Security benefit expenditure,  $Tr_{SS,t}$ , equals

$$Tr_{SS,t} = \sum_{i=20}^{120} \int_{E \times A \times B} tr_{SS,t}(i, b_i) dX_t(\mathbf{s}_i). \quad (14)$$

The laws of motion of the trust funds and the rest of government net wealth are both normalized by productivity growth and long-run population growth:

$$W_{S,t+1} = \frac{1}{(1+\xi)(1+\nu)} \{(1+r_t)W_{S,t} + T_{PO,t} - Tr_{SS,t}\}, \quad (15)$$

$$W_{G,t+1} = \frac{1}{(1+\xi)(1+\nu)} \{(1+r_t)W_{G,t} + T_{I,t} + T_{PH,t} - C_{G,t}\}. \quad (16)$$

**Definition Recursive Competitive Equilibrium:** A series of factor prices, accidental bequests, government policy variables, and the parameters  $\varphi_s$  of government policy functions,

$$\Omega_t = \{r_s, \bar{w}_s, q_s, W_{S,s+1}, W_{G,s+1}, C_{G,s}, \varphi_s\}_{s=t}^{\infty};$$

the value function of households,  $\{v(\mathbf{s}_i, \mathbf{S}_s; \Psi_s, \Phi_s)\}_{s=t}^{\infty}$ ; the decision rule of households,

$$\{\mathbf{d}(\mathbf{s}_i, \mathbf{S}_s; \Psi_s, \Phi_s)\}_{s=t}^{\infty} = \{c_i(\mathbf{s}_i, \mathbf{S}_s; \Psi_s, \Phi_s), h_i(\mathbf{s}_i, \mathbf{S}_s; \Psi_s, \Phi_s), a_{i+1}(\mathbf{s}_i, \mathbf{S}_s; \Psi_s, \Phi_s)\}_{s=t}^{\infty};$$

and the measure of households,  $\{x_s(\mathbf{s}_i)\}_{s=t}^{\infty}$ , are in a recursive competitive equilibrium if, in every year  $s = t, \dots, \infty$ , each household solves the utility maximization problem (1) – (2) taking  $\Psi_t$  and  $\Phi_t$  as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, (7) – (10) hold; the government policy rules satisfy (11) – (16); and the goods market clears.

## 2.1 Calibration

We calibrate the model to simulate the steady state, because in this paper we are interested in the properties of the model and not in the evolution of the economy responding to a particular policy. The time of the model is one year and all households are assumed

Table 1: Parameters

Time preference parameter	$\beta$	0.986
Share parameter for consumption	$\alpha$	0.467
Coefficient of relative risk aversion	$\gamma$	2.0
Capital share of output	$\theta$	0.300
Depreciation rate of capital stock	$\delta$	0.047
Long-term real growth rate	$\xi$	0.018
Long-term population growth rate	$\nu$	0.01
Total factor productivity	$A$	0.949

to be two-earner married couples of the same age. Table 1 summarizes the parameter choices. The calibration strategy is similar to Nishiyama (2004) and Nishiyama and Smetters (2005), but some parameters are revised.

$\beta$  is chosen so that the capital-output ratio is 2.74. The share parameter for consumption  $\alpha$  is chosen so that the average annual working hours are 3,414 hours.<sup>8</sup> The maximum working hours  $h^{\max}$  are set to be 8,760 for all households, which equals 12 hours per day per person  $\times$  365 days  $\times$  two persons. The coefficient of relative risk aversion  $\gamma$  is assumed to be 2.0.<sup>9</sup> The capital share parameter  $\theta$  of the production function and depreciation rate  $\delta$  are calculated from macroeconomic statistics as in Nishiyama (2004). Total factor productivity  $A$  is chosen so that the wage rate  $\bar{w}$  equals unity.

The model uses a constant relative risk aversion (CRRA) function defined as follows:

$$u_i(c_i, h_i) = \frac{\left\{ \left( \left( 1 + \frac{n_i}{2} \right)^{-\zeta} c_i \right)^\alpha (h^{\max} - h_i)^{1-\alpha} \right\}^{1-\gamma}}{1 - \gamma} \quad (17)$$

where  $\gamma$  is the coefficient of relative risk aversion,  $n_i$  is the number of dependent children,  $\zeta$  is the consumption adjustment parameter, and  $h_i^{\max}$  is the maximum working hours.

<sup>8</sup>This is average annual working hours of married couples aged between 20 and 64 in the 2001 Survey of Consumer Finances (SCF), and the annual working hours in the model are the sum of the working hours of a husband and a wife.

<sup>9</sup>There is no strong consensus about the coefficient of relative risk aversion. For the estimates of  $\gamma$  in previous literature, see Auerbach and Kotlikoff (1987) and Prescott (1986). Citing these two papers, Huggett (1996) used  $\gamma$  of 1.5 and 3.0. More recently, Gourinchas and Parker (2002) estimated  $\gamma$  varying between 0.5 and 1.4, and Laitner (2003) estimated  $\gamma$  at 2.3 or 2.0.

The numbers of dependent children by parents' age are calculated from the Panel Study of Income Dynamics (PSID) 1993 Family Data.

**Working Ability ( $e$ ).** The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 2001 SCF. The average hourly wage of a married couple (family members #1 and #2 in SCF) used for the calibration is calculated by

$$\text{Hourly Wage} = \frac{\text{Regular and Additional Salaries (\#1 + \#2)} + \frac{1}{2} \times \text{Imputed Payroll Tax}}{\max \{\text{Working Hours (\#1 + \#2)}, 2080\}}.$$

The payroll tax for Social Security paid by its employer is also imputed and included to the household's labor income. Table 2 shows the eight discrete levels of working abilities of five-year age cohorts. We calculate the working ability of each age from 20 to 79 by using a shape-preserving cubic spline interpolation.<sup>10</sup>

**Markov Transition Matrices.** The transition probability from one working-ability class to another is calculated directly from the Panel Study of Income Dynamics (PSID). Separate transition matrices were constructed for four age groups—20-29, 30-39, 40-49, and 50-59—to capture the possibility that the probabilities themselves might change over the lifecycle. (For households aged 60 or older, we used the matrix for ages 50-59.) Table 3 shows the transition probabilities used in the model.

To give an idea of the persistence and variance of the labor income shocks, we calculate the AR(1) representation of the process for labor income by age described by the transition probability matrices and the working abilities in tables 2 and 3.<sup>11</sup> The parameters are shown in table 4.

**Borrowing Constraints.** We set the minimum level of assets to a value that makes 19% of the households to be liquidity constrained as reported for the US by Japelli

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<sup>10</sup>In the calibration, the numbers in the table are multiplied by 1.038 to convert the hourly wages in 2000 into those in 2001.

<sup>11</sup>A more detailed parameterization of the labor income process would account for the permanent and transitory labor income shocks implied by the transition matrices and the working ability values presented above. We remain agnostic about the precise data generating process for the time series of the labor income and, instead, we calculate directly the transition matrices for the Markov process.

Table 2: Working Abilities of a Household (in 2000 US\$ per Hour)

Percentile		Age cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
$e^1$	0-20th	3.89	5.47	6.86	6.01	7.43	5.73
$e^2$	20-40th	8.35	10.11	12.38	12.27	13.90	13.14
$e^3$	40-60th	10.28	14.04	16.46	16.96	18.76	18.47
$e^4$	60-80th	12.31	17.30	21.87	22.57	25.79	25.71
$e^5$	80-90th	17.47	21.58	29.37	30.19	35.37	35.56
$e^6$	90-95th	22.17	27.21	33.96	46.92	48.30	54.59
$e^7$	95-99th	29.43	36.60	43.76	81.75	96.44	97.48
$e^8$	99-100th	42.31	62.29	182.78	327.65	262.03	284.00

Percentile		Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
$e^1$	0-20th	5.00	4.31	0.58	0.00	0.00	0.00
$e^2$	20-40th	13.99	12.10	7.93	0.57	0.00	0.00
$e^3$	40-60th	20.95	17.07	13.50	8.42	0.66	0.14
$e^4$	60-80th	29.13	26.39	21.26	14.99	7.59	7.07
$e^5$	80-90th	40.89	36.36	32.83	24.60	17.18	18.31
$e^6$	90-95th	54.11	53.44	48.40	38.49	30.02	25.86
$e^7$	95-99th	91.67	101.96	100.57	63.92	47.66	30.25
$e^8$	99-100th	282.18	342.00	613.42	279.29	256.40	116.25

Source: Nishiyama and Smetters (2005) from the 2001 SCF data.

(1990). This value is approximately equal to the 0.1% of the maximum level of assets that the agents hold in the model (this is equal to  $0.001\bar{a}$ ). The gini coefficient for wealth in the calibrated model is 0.79, very close to the US Gini coefficient of 0.78 in 2001<sup>12</sup>

**The Social Security System.** The tax rate levied on both employers and employees for OASDI is 12.4 percent, and the tax rate for HI is 2.9 percent. In 2001, employee wages above \$80,400 were not taxable for OASDI. Social Security benefits are calculated from each worker’s Average Indexed Monthly Earnings (AIME),  $b_{65}/12$ , and the replacement rate schedule in the United States. (90% for AIME less than \$561, 32% for AIME between \$561 and \$3,381 and 15% for AIME above that level). The average historical earnings  $b_i$  are the approximation of the Average Monthly Indexed Earnings (AIME)

<sup>12</sup>See “What’s the Difference? Evidence on the Distribution of Wealth, Health, Life Expectancy and Health Insurance Coverage”, Arthur B. Kennickell, Federal Reserve Board Paper, October 2008 and “A Rolling Tide: Changes in the Distribution of Wealth in the U.S., 1989-2001” Arthur B. Kennickell, March 2003

Table 3: Markov Transition Matrices

Working ability	$e^1$	$e^2$	$e^3$	$e^4$	$e^5$	$e^6$	$e^7$	$e^8$
Age20-29								
$e^1$	0.2552	0.0895	0.0227	0.0179	0	0.0042	0.0001	0.0003
$e^2$	0.3714	0.3923	0.1862	0.0408	0.0241	0.0048	0.0042	0.0029
$e^3$	0.0291	0.2858	0.3216	0.0882	0.038	0.0153	0.0088	0.0048
$e^4$	0.2615	0.1163	0.2462	0.4285	0.1314	0.0342	0.0104	0.0118
$e^5$	0.0012	0.0373	0.1084	0.2947	0.4576	0.2385	0.0756	0.0464
$e^6$	0	0.0432	0.0898	0.0784	0.2016	0.4084	0.2322	0.0875
$e^7$	0.044	0.0248	0.0005	0.0202	0.0831	0.1902	0.4594	0.2499
$e^8$	0.0376	0.0108	0.0246	0.0313	0.0642	0.1044	0.2093	0.5964
Age 30-39								
$e^1$	0.5392	0.0645	0.0281	0	0.0015	0	0.0016	0
$e^2$	0.3209	0.4869	0.1512	0.0484	0.0116	0.0069	0	0.0061
$e^3$	0.0384	0.2624	0.3798	0.1182	0.0281	0.011	0.0006	0.0018
$e^4$	0.0379	0.083	0.2946	0.4581	0.1526	0.0154	0.0077	0.0031
$e^5$	0.0334	0.0621	0.0907	0.3065	0.5411	0.2018	0.0341	0.033
$e^6$	0	0.0007	0.0084	0.0348	0.2024	0.5233	0.2001	0.0546
$e^7$	0.0302	0	0.0309	0.0068	0.0413	0.1796	0.5587	0.2078
$e^8$	0	0.0404	0.0163	0.0272	0.0214	0.062	0.1972	0.6936
Age 40-49								
$e^1$	0.4394	0.0615	0.0424	0	0.0022	0	0.0029	0
$e^2$	0.2502	0.4942	0.2094	0.0593	0.0027	0.002	0.002	0
$e^3$	0.13	0.2486	0.3653	0.1055	0.0311	0.0117	0.0052	0.0017
$e^4$	0.0786	0.0777	0.1427	0.4644	0.1576	0.0339	0.0142	0.007
$e^5$	0.0475	0.0493	0.1385	0.3155	0.5587	0.1996	0.0261	0.011
$e^6$	0	0.0354	0.0512	0.0217	0.1833	0.5429	0.2078	0.0352
$e^7$	0.0543	0.0086	0.0089	0.0145	0.043	0.152	0.5571	0.234
$e^8$	0	0.0247	0.0416	0.0191	0.0214	0.0579	0.1847	0.7111
Age 50-59								
$e^1$	0.5786	0.0866	0.015	0	0	0	0	0
$e^2$	0.2386	0.4079	0.2444	0.0215	0.0169	0.0102	0	0.0076
$e^3$	0.1701	0.2026	0.3345	0.169	0.019	0.0123	0	0.0015
$e^4$	0.0127	0.2008	0.2605	0.4774	0.122	0.0197	0.0027	0.011
$e^5$	0	0.0213	0.0898	0.2608	0.6075	0.2043	0.0189	0.0121
$e^6$	0	0.0409	0.0429	0.0352	0.1651	0.5353	0.2117	0.0514
$e^7$	0	0.021	0.0127	0.0085	0.0525	0.1743	0.5452	0.2164
$e^8$	0	0.0189	0.0002	0.0276	0.017	0.0439	0.2215	0.7

Source: Nishiyama and Smetters (2005) from the PSID data.

multiplied by 12 and used to calculate the household's Social Security benefits in the model. In the model, survivors' benefits and disability insurance are simply assumed to be proportional to the benefits for old-age workers. This enters as a de facto transfer to households. We adjust the statutory old age benefits in the model by a factor of 1.448 so that the total amount of benefit payments equals the total amount of taxes collected.

**Federal, State and Local Income Taxes.** Federal income tax rates are assumed to be at the level they have been after the 2001 and 2003 tax cuts. Table 5 shows the statutory marginal tax rates during that year. The statutory federal income tax is multiplied by an adjustment factor equal to 0.795 so that the federal income tax revenue

Table 4: AR(1) representation of the earnings ability process ( $e_{it} = \rho e_{i,t-1} + \epsilon_{it}$ )

Age	$\rho$	$\sigma_\epsilon$
20-29	0.7178	0.4111
30-39	0.8018	0.4161
40-49	0.8029	0.4503
50-59	0.8661	1.02

Table 5: Marginal Individual Income Tax Rates in 2007 (Married Household, Filed Jointly)

Taxable Income (2001 US\$)		Marginal Income Tax Rate (%)
\$0	–	10.0
\$13,367	–	15.0
\$54,409	–	25.0
\$109,758	–	28.0
\$167,284	–	33.0
\$298,694	–	35.0

The standard deduction is \$9,139 and exemption per person is \$2,904.

(including corporate income tax) is 11.2% of GDP as in 2007.<sup>13</sup> The total state and local income tax rate is assumed to be a flat 4%.

### 3 Quantifying the Frisch Elasticity at the individual level

In this section we lay down a theoretical framework to analyze two types of intertemporal elasticity of labor supply. The first—the Frisch elasticity—measures the response of hours of work to anticipated wage changes, holding the marginal utility of wealth constant. The second measures the response of hours of work to unanticipated wage changes. Both elasticities are relevant to the analysis of tax and benefit reforms, and their relative importance depends on whether the reforms favor wealth or intertemporal

<sup>13</sup>Additionally, capital income is multiplied by 0.767 before applying income taxes, reflecting the lower effective tax rate on capital income.

substitution effects differently. We discuss the empirical challenges faced in estimating the elasticities and the issues that arise when we want to map the results from the model to the empirical estimates. We quantify the Frisch elasticity directly from the model, not accounting for borrowing constraints or uncertainty, and we quantify the same elasticity accounting for the borrowing constraints using data from a simulated panel. We also estimate from the model the labor supply response to unanticipated wage changes and quantify the biases induced by the omission of wage shocks and wage uncertainty. In this section, we do not include considerations about participation constraints in the estimation procedure and focus in the Frisch elasticity calculated at the intensive margin.

### 3.1 Frisch Elasticity derived from the theoretical individual problem

The optimality conditions for the consumer problem are

$$u_{c_t} = \lambda_t \tag{18}$$

$$u_{h_t} = \lambda_t(\bar{w}_t e_t + \frac{\partial \tau(\bar{w}_t e_t h_t)}{\partial h}) + \mu = \lambda_t \bar{w}_t e_t (1 - \tau') + \mu = \lambda_t w_t + \mu \tag{19}$$

$$\lambda_t = E_t \left( \frac{1 + r_{t+1}}{1 + \rho} \right) \lambda_{t+1} + \kappa_t \tag{20}$$

Where  $\lambda_t$  represents the marginal utility of wealth in period  $t$  normalized by the steady state per-capita growth rate, this is,  $\lambda = \lambda' * \frac{1}{1+\mu}$ ,  $\kappa_t$  is the lagrange multiplier associated with the borrowing constraint in (2) which would be equal to zero when the household does not face liquidity constraints and different from zero otherwise.  $\mu$  is the lagrange multiplier associated with the participation constraint that takes a value of zero when the household decides not to work.

$\tau$  is the effective tax paid by the household. As stated in equation (3), this comprises the effective progressive income taxes, the income tax for the Old-Age, Survivors, and Disability Insurance (OASDI), the payroll tax for the Hospital Insurance (HI), the accrued social security benefits and the progressive income taxes, and can be approximated

as a function of  $w$  and  $h$ . For simplicity in the exposition, we summarized them up in one term, and we denote by  $w$  the after-tax wage rate. We ignore second order effects of the change of hours on the effective tax rate because all the tax functions are linear and the jumps are at specific points that have negligible influence in the total elasticity.

From equations (18) and (19), we can derive the Frisch elasticity of labor supply, that is, how the number of hours responds intertemporally to changes in the wage rate, holding the marginal utility of wealth constant, if neither the borrowing nor the participation constraint bind and if we ignore uncertainty. Dropping the time subscript, the expression for the Frisch elasticity without considering borrowing constraints is as follows:<sup>14</sup>

$$\left. \frac{dh}{dw} \frac{w}{h} \right|_{\lambda} \Big|_{\text{no constraint}} = \frac{u_h u_{cc}}{u_{hh} u_{cc} - u_{hc}^2} \frac{1}{h} \quad (21)$$

Equation (21) implies that, if there is no change in consumption when the number of hours changes ( $u_{ch} = 0$ ), the Frisch elasticity is given by  $(u_h/u_{hh})(1/h)$ . That happens, for example, when the utility function is separable in labor and consumption, as was the case in many previous studies.<sup>15</sup> When consumption and labor are non separable, households can substitute consumption for labor to attain the same level of utility, affecting their response to changes in wages. The non-separability of consumption and labor in the utility function thus means that our model allows for substitution between consumption of goods and consumption of leisure, and given a set of calibrated parameters, the macroeconomic behavior will be different. Moreover, this specification allows us to better match the targets in the aggregate economy and better track the behavior of labor supply.

To quantify the Frisch elasticity without considering participation or borrowing constraints (by holding the marginal utility of wealth constant we ignore all future information about wages and interest rates, this is, we ignore uncertainty), we use equation

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<sup>14</sup>The calculation of the Frisch elasticity using the after tax wage implies that the expression for the Frisch elasticity has to be divided by  $1 - \tau$  as equation 19 implies

<sup>15</sup>The assumption that labor and consumption are additively separable has been important to many previous empirical estimates, because it implies that the Frisch elasticity can be calculated directly using the parameters of the utility function (see for example Altonji (1986), Heckman and MaCurdy (1982, in the corrigendum to their initial article in 1980) and MaCurdy (1981))

(21) and the model's utility function in equation (17) and we arrive at an expression for the Frisch elasticity of labor supply without considering borrowing or participation constraints or uncertainty in wages:

$$\frac{dh}{dw} \frac{w}{h} \Big|_{\lambda} = \frac{h_t^{max} - h_t}{h_t} \left\{ \frac{1 - \alpha(1 - \gamma)}{\gamma} \right\} \quad (22)$$

From the calibration, we use  $h^{average} = 3414$ ,  $h^{max} = 8760$  representing 2 persons working 12 hours each,  $\gamma = 2.0$ ,  $\alpha = 0.465$ , and a marginal tax rate of 21%. With those values, the intertemporal Frisch elasticity of labor supply (which we denote by  $\eta$ ) obtained in this fashion has a value of  $\eta = 1.14$ . This implies in turn a value for the elasticity of leisure to changes in wages (denoted by  $\eta^l$ ) of  $\eta^l = \eta * \frac{h^{average}}{h^{max} - h^{average}} = 0.73$

### 3.2 Estimates of the Labor Supply Responses to Anticipated (Frisch) and Unanticipated Changes in Wages from a Simulated Panel

In practice, when using real data to estimate the elasticity in equation (22), the econometrician has to control for borrowing constraints and for uncertainty about future wages and interest rates. The borrowing constraints reduce the response of hours of work to a change in wages, and the presence of uncertainty introduces a different type of intertemporal labor supply response, which we measure later as the response of hours of work to unanticipated changes in wages.<sup>16</sup>

Uninsurable wage risk, accidental bequests, a social security system that provides an additional source of insurance, and borrowing constraints are crucial features of the stochastic OLG model used in this paper to adequately represent the US economy. Agents live through many periods and face borrowing constraints, uncertain time of death, and idiosyncratic shocks to working ability. Bad wage shocks can slam households onto borrowing constraints that effectively break the intertemporal links across periods in

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<sup>16</sup>This type of intertemporal labor supply elasticity exists only if uncertainty is present, because idiosyncratic shocks cannot be anticipated by the households, and they trigger this specific response.

the model. When the constraints bind, households in the model effectively become static, and the lifetime period is broken up into a series of shorter periods. All those factors are expected to attenuate the labor supply response to policy changes, but quantifying that attenuation is not easy.

To overcome those difficulties, we use a general method that can be summarized in two steps. First, we generate a simulated panel with the OLG model calibrated to the US economy with one of the targets being the number of borrowing-constrained agents. We generate another panel using the same economy but without any binding borrowing constraints. Second, we estimate for both panels two types of intertemporal elasticities using several empirical methodologies that have already been used with real data. The elasticities we estimate are the labor supply response to anticipated changes in wages, holding the marginal utility of wealth constant (Frisch elasticity), and the labor supply response to unanticipated changes in wages.

The results from the panel where borrowing constraints never bind show the effect of all the uncertainty on the Frisch elasticity, and the comparison with the results from the panel where borrowing constraints bind shows the effect of borrowing constraints on the Frisch elasticity. By calculating the responses to unanticipated changes in wages we explore the biases induced by not considering that particular source of uncertainty in the estimates of the Frisch elasticity.

To clarify the method, we start with the basic estimation equation that uses the first order conditions (18) and (19) and the specific utility function which is nonseparable between consumption and leisure described in equation (17):

$$\ln(h^{max} - h_{it}) = c - \eta^l \ln w_{it} - \left( \frac{\eta^l}{1 - \alpha(1 - \gamma)} \right) \ln \lambda_{it} \quad (23)$$

where  $c$  is a constant and  $\eta^l$  represents the intertemporal elasticity of leisure, which is equal to  $\eta^l = \frac{h_{it}}{h^{max} - h_{it}} \eta$ , being  $\eta$  the Frisch elasticity. The substitution between consumption and labor affects the estimates through the term  $1 - \alpha(1 - \gamma)$ .<sup>17</sup>

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<sup>17</sup>Considering participation constraints would add the term  $-\eta^l \frac{\mu_{it}}{\lambda_{it} w_{it}}$

Equation (23) can be used to analyze the effect of borrowing constraints, participation constraints and uncertainty in the Frisch elasticity of labor supply, in the same spirit as Domeij and Floden (2006). We can express that equation in differences, use equation (20), and define the expectation error over the marginal utility of wealth as  $\epsilon_{it+1} = \lambda_{it+1} - E\lambda_{it+1}$  to arrive at an approximated expression for the change in the marginal utility of wealth:  $\Delta \ln \lambda_{it} = (\rho - E_{t-1}r_t) - \frac{\kappa_{it}}{\lambda_{it}} + \sum_{j=1}^{\infty} (\frac{1}{j})(\frac{\epsilon_{it+1}}{\lambda_{it+1}})$ .<sup>18</sup> This way we arrive at the following new version of equation (23):

$$\begin{aligned} \Delta \ln(h^{max} - h_{it}) &= c - \eta^l \Delta \ln w_{it} \\ &- \left( \frac{\eta^l}{1 - \alpha(1 - \gamma)} \right) \left\{ -(E_{t-1}r_t - \rho) - \frac{\kappa_{it}}{\lambda_{it}} + \sum_{j=1}^{\infty} \left( \frac{1}{j} \right) \left( \frac{\epsilon_{it+1}}{\lambda_{it+1}} \right) \right\} \end{aligned} \quad (24)$$

Through the term  $\frac{\kappa_{it}}{\lambda_{it}}$ , equation (25) shows the negative effect of borrowing constraints  $\kappa_{it}$  in the Frisch elasticity of labor supply  $\eta = \frac{h_{it}}{h^{max} - h_{it}} \eta^l$ , and through the term  $\sum_{j=1}^{\infty} (\frac{1}{j})(\frac{\epsilon_{it+1}}{\lambda_{it+1}})$  it shows the effects of uncertainty. It can be argued that the expected change in wages is negatively correlated with the marginal utility of wealth because the marginal utility of wealth decreases by the effect of the concavity of the utility function. Then, if the marginal utility of wealth is not controlled directly, we would have a negative correlation between the terms  $E_{t-1} \Delta \ln w_{it}$  and  $\frac{\kappa_{it}}{\lambda_{it}}$ , inducing a negative bias in the estimate of  $\eta$ , the Frisch elasticity of labor supply.<sup>19</sup>

Researchers face three important problems when they try to estimate equations like equation (23): First, neither the marginal utility of wealth nor the Lagrange multiplier associated with the borrowing constraints is observed by the econometrician. Second, the change in expected log of wages is not observed. Third, the change in expected log of wages is likely to be correlated with the marginal utility of wealth or with the change in tastes for leisure,  $Z_{it}$ . A widely applied option is to use instrumental variables techniques for that term.<sup>20</sup>

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<sup>18</sup>This expression uses a first order approximation of the left hand side of (20) around  $\mu_{it} = 0$  and a n-th order approximation of the right hand side of (20) around  $\epsilon_{it+1} = 0$

<sup>19</sup>Consideration of participation constraints would add the term  $-\eta^l \Delta \left( \frac{\mu_{it}}{\lambda_{it} w_{it}} \right)$

<sup>20</sup>Keane (2006) also points out that it is difficult to develop a good instrument to predict the expected change in log wages, and commonly used variables, such as age and education, could be associated with

We use equation (23) to calculate the Frisch elasticity instrumenting for the change in the log of wages using lagged wages (Altonji (1986)), life cycle instruments (MaCurdy (1981)) and using Pistaferri (2003) approach. To estimate the effect of the approximation error vs the effect of the borrowing constraints, we generate a panel where borrowing constraints do not bind (this is, the term  $\frac{\kappa_{it}}{\lambda_{it}}$  does not appear) and compare the results with those generated using a panel where borrowing constraints bind. We then narrow further the analysis to calculate the effect of a particular source of uncertainty, namely the effect of wage shocks and wage uncertainty, which in the literature has been estimated as the labor supply response to unanticipated changes in wages using Pistaferri (2003) methodology because it accounts directly for variances and shocks, and it controls for changes in the marginal utility of wealth. Given that we can generate exact data and calculate expectations from the model, our estimate is free of measurement errors that arise when people misjudge their expected income or consumption or misreport current values.

The Pistaferri (2003) method uses the usual consumer problem as described in equation (18), takes a first order approximation of the equation (23) and ignores borrowing and participation constraints. Using those steps, we can rewrite 23 as:

$$\begin{aligned} \Delta \ln(h^{max} - h_{it}) &= c - \eta^l \Delta \ln w_{it} + \left( \frac{\eta^l}{\alpha(1-\gamma) - 1} \right) (E_{t-1} r_t - \rho) \\ &\quad - \left( \frac{\eta^l}{\alpha(1-\gamma) - 1} \right) (\ln \lambda_{it} - E_{t-1} \ln \lambda_{it}) \\ &\quad + \left( \frac{\eta^l}{\alpha(1-\gamma) - 1} \right) \ln E_{t-1} (e^{\ln \lambda_{it} E_{t-1} \ln \lambda_{it}}) \end{aligned} \quad (25)$$

In order to get an equation that we can estimate, we use the following assumptions as Pistaferri (2003): (i) The wage process is parameterized as  $\ln w_{it} = \Delta X'_{it} \sigma + \ln w_{it-1} + \zeta_{it}$ , where  $X$  is a vector of deterministic variables affecting wages (age, education) and  $\zeta$  is the innovation in the wage rate  $\zeta = \ln w_{it} - E_{t-1} \ln w_{it-1}$ . (ii) The log of the marginal utility of wealth ( $\lambda$ ) is parameterized as  $\left( \frac{\eta^l}{\alpha(1-\gamma)-1} \right) \ln \lambda_{it} = \gamma_a + \sum_{\tau=0}^T \gamma_\tau E_t(\ln w_{it+\tau}) + \nu_i$ , where  $\eta^l$  is the intertemporal elasticity of leisure, and  $\alpha(1-\gamma)$  captures the substitution preference shifts or with elements present in the error term, again introducing bias in the estimates.

of consumption for leisure.  $\left(\frac{\eta^l}{\alpha(1-\gamma)-1}\right)$  is the elasticity of leisure with respect to the marginal utility of wealth. From here, we can observe that  $\Gamma = \sum_{\tau=0}^T \gamma_\tau$  is the wealth effect of a parametric permanent shift in the wage profile. With those assumptions, we use the general theoretical formula for the Frisch elasticity described in equation (21) and incorporate the interest rate and the discount rate in the constant term (they are the same for all households). After taking conditional expectations, we arrive to the following equation:

$$\Delta \ln(h^{max} - h_{it}) = c - \eta^l E_{t-1} \Delta \ln w_{it} - v Var_{t-1}(\zeta_{it}) - (\eta^l + \Gamma) \zeta_{it} \quad (26)$$

$\eta^l$  represents the value of the intertemporal elasticity of leisure, which measures the change in leisure (or, conversely, the change in labor supply) in reaction to expected changes in wages, holding the marginal utility of wealth constant. The other parameters of interest are  $\Gamma$ , which measures the change in leisure to unexpected changes in wages, and  $v$ , which measures the change in leisure in response to wage risk and is equal to  $v = -\Gamma^2 / [2(\frac{\eta^l}{\alpha(1-\gamma)-1})]$ .<sup>21</sup> Note that with this transformation, we don't have to use instruments on  $\Delta \ln w_t$ , and instead can use directly the expected wage if we have data. The variables generated using the model are hours ( $h_{it}$ ), expected change in the log of wages ( $E_{t-1} \Delta \ln w_{it}$ ), and innovation in the wage rate  $\zeta = \ln w_{it} - E_{t-1} \ln w_{it-1}$ . Because our data were generated by simulation from a life-cycle model, we have the advantage over most empirical studies of having the precise variables we need to estimate the parameters of interest and no measurement error problems to deal with.

Table 6 shows the results of the estimations using the simulated data when borrowing constraints bind (making 19% of households liquidity constrained) and when they do not bind. Our baseline panel is the one where borrowing constraints bind. We analyze the responses at the intensive margin (when the labor supply is more than zero hours) for households whose age is less than 65 to align the sample selection with similar empirical studies. We estimate the intertemporal substitution of leisure, which can be mapped

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<sup>21</sup>The literature refers to anticipated changes in wages as “evolutionary,” and to unanticipated changes as “parametric.”

Table 6. Leisure elasticity estimates using the simulated panel

	Instruments			Pistaferri (2003)
	lagged wages (Altonji) $\Delta \ln w_{t-1}, \ln w_{t-1}$	life cycle (MaCurdy) $age, age^2$	$E_{t-1} \ln w_t$	
	Budget constraint binds			
$\eta^l$	0.49 (0.00)	0.12 (0.00)	0.30 (0.00)	0.29 (0.00)
	Budget constraint does not bind			
$\eta^l$	0.56 (0.00)	0.20 (0.00)	0.33 (0.00)	0.32 (0.00)
Bias	0.24	0.61	0.44	0.44
Bias due to				
approximation error	0.17	0.53	0.40	0.41
borrowing constraints	0.07	0.08	0.04	0.03

directly into the Frisch elasticity through the formula  $\eta^l = \eta * \frac{h^{average}}{h^{max} - h^{average}}$ . Using the utility function and the values for the maximum number of hours and for average hours, the theoretical leisure elasticity is 0.73 (Frisch 1.14). The table shows the results using lagged wages and life-cycle indicators (age, age squared) as instruments for the expected wages, and the results obtained by applying equation (26) as in Pistaferri (2003).

Several points stand out from table 6. First, the empirical estimates from the baseline panel are lower by a factor between 1.5 (life cycle instruments) and 6 (lagged wages) than the theoretical value of the leisure elasticity. Life cycle instruments perform the worst because they do not capture any kind of uncertainty. When we use the same life cycle instruments but control for wage risk, we obtain estimates that are 3 times bigger than the original estimate (Table 7).

The second important point is that most of the bias comes from the approximation error which, in this case, accounts for all higher order terms. The borrowing constraints account for between 7% and 29% of the biases depending on the method, and the approximation error accounts for between 71% and 93%. This result is in contrast with Domeij and Floden, who find the opposite. The differences come mainly from the type of model used and from the sources of insurance that we allow the agents to have: besides

Table 7. Biases induced on the leisure elasticity by ignoring wage risk and wage uncertainty using life cycle instruments (*age*, *age*<sup>2</sup>)

$\eta^l$	0.34 (0.01)	0.34 (0.00)	0.12 (0.00)	0.12 (0.01)
Controls				
wage risk	yes	yes	no	no
unexpected wage shocks	yes	no	yes	no

Table 8. Effect of borrowing constraints by level of wealth

	$a_{t+1} \leq a^{min}$	$a_{t+1} > a^{min}$	$a_{t+1} \leq 0.1\bar{a}$	$a_{t+1} \geq 0.1\bar{a}$
$\eta^l$	0.06 (0.00)	0.47 (0.00)	0.20 (0.00)	0.51 (0.00)

$a^{min} = 0.001\bar{a}$  where  $a^{min}$  is the minimum level of assets, and  $\bar{a}$  is the maximum level of assets a household holds in this economy

self-insurance through savings, agents in our model have access to social security and receive accidental bequests, and they plan in a lifecycle framework.

To allow closer comparisons with the empirical literature, our results can be transformed into Frisch elasticity estimates; with that specification, their values are between 0.19 and 0.77. Comparisons are striking: our results overlap with the range of 0.1-0.45 given by MaCurdy (1981), they account for the central value of 0.2 in Pencavel's 1986 survey, they are very similar to the range of 0.24-0.79 given by Angrist (2005) and the higher end is close to Pistaferri's (2003) estimate of 0.7. In making those comparisons, it's important to notice that we generate data on wages and expected wages directly, that we isolate the estimates from unobserved taste shifts, that we can estimate the Frisch elasticity using wages instead of earnings, that our data do not have measurement error and that we can control the macroeconomic conditions, where unemployment and other demand-side variables can alter the labor supply responses.

Table 8 shows how the degree of borrowing constraints matter in this economy. For strictly borrowed constrained agents the leisure elasticity is 0.06 and for other agents this

Table 9. Effect of wage risk and wage uncertainty on the leisure elasticity

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$\eta^l$	0.3064	0.3064	0.2925	0.2925
	(0.0004)	(0.0004)	(0.0004)	(0.0001)
$v$	0.0007	0.0007		
(wage risk)	(0.0000)	(0.0000)		
$\eta^l + \Gamma$	0.0001		0.0002	
(unexpected changes in wages)	(0.0000)		(0.0000)	

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Table 10. Wealth effects by age

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age	< 30	30 – 40	40 – 50	50 – 60	60 – 70
$\Gamma$	-0.31	-0.39	-0.39	-0.35	-0.19
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

---

elasticity is 0.47, a large difference. The same is true for agents that are less borrowing constrained (the ones that hold 10% of the maximum level of assets) compared to the ones with assets above that threshold (0.20 vs. 0.51).

We explore further the role of wage uncertainty and wage shocks as part of the approximation error. Table 9 presents the estimates of the leisure intertemporal response to wage shocks and wage risk following Pistaferri’s (2003) method. The labor supply response in the model to unanticipated wage changes is very small (0.0007). That value is below previous empirical estimates. For example, MaCurdy (1981) has 0.08 and Pistaferri (2003) has 0.51. At the same time, our estimate implies a wealth effect of -0.31, which significantly ameliorates the intertemporal substitution effect in response to wage changes. The wealth effect estimate in our model is somewhat higher than Pistaferri’s estimate of -0.2 and significantly higher than MaCurdy’s estimate of -0.07. As expected, wealth effects decrease in importance with age (Table 10) as households become wealthier.

## 4 Quantifying the Frisch elasticity using macro data (cross-sectional aggregation)

Using the same simulated data, we aggregate the values for hours and wages (to calculate the wage rate) and calculate the participation rate by age. We calculate the macro Frisch elasticity at the extensive margin (work or not work) by regressing the log of the participation rate on age and the log of the wage rate. We obtain a value of 0.17. This number is close to the estimates obtained with micro data and reviewed in Chetty et al (2010), which are between 0.18 and 0.42.

We calculate the macro Frisch elasticity at the intensive margin by regressing the change in the log of aggregate hours on age and the change in the log of the wage rate. We obtain a value of 1.4, 20% higher than the theoretical value and up to 7 times bigger than the empirical micro estimates. This estimate, however is biased upwards because of aggregation problems.

Start with equation (25) (and ignore borrowing or participation constraints). The aggregate version of that equation would be:

$$\Delta \sum \ln(h_{it}) = K + \Delta \sum \ln(w_{it}) \quad (27)$$

With aggregate data, however, that equation would be estimated as

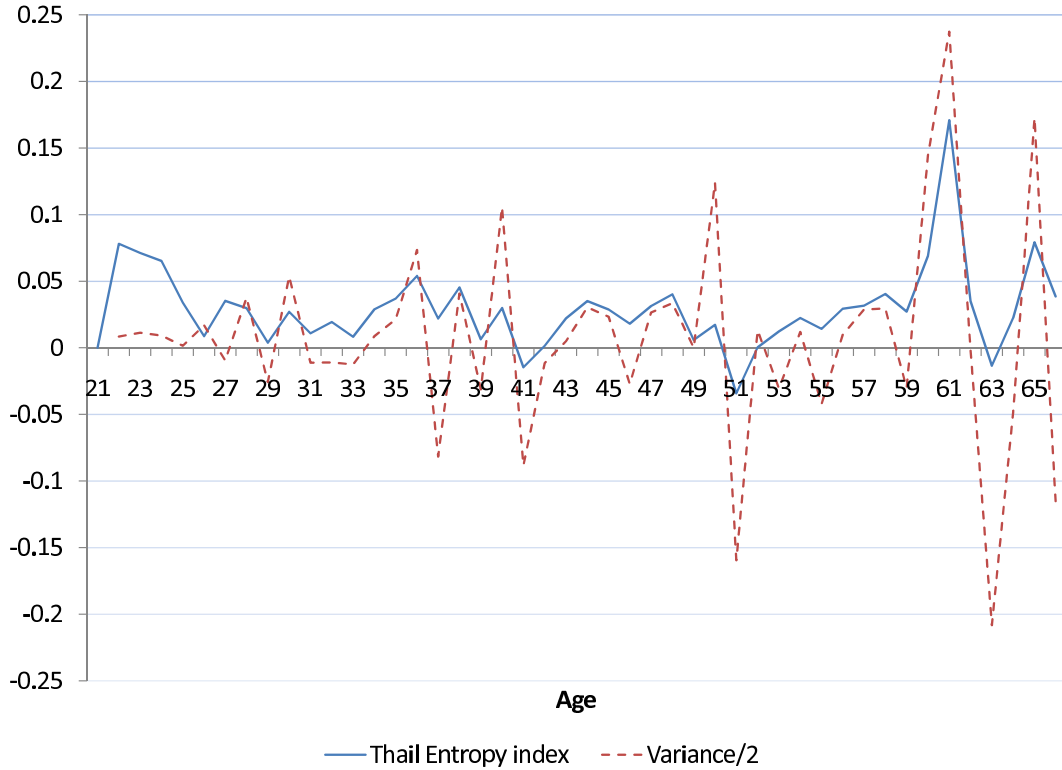
$$\Delta \ln(\sum h_{it}) = K + \Delta \ln(\sum w_{it}) \quad (28)$$

Attanasio and Weber (1993) show that, in the case of consumption, the difference between equation (28) and equation (27) is given by the Thail's entropy index:

$$\Delta \ln(\sum h_{it}) - \Delta \sum \ln(h_{it}) = \Delta \ln\left(1 + \frac{1}{2}\mu_{2t} + \frac{1}{6}\mu_{3t} + \frac{1}{24}\mu_{4t}\right) \quad (29)$$

where  $\mu_{it}$  is the i-th cross-sectional moment of  $\Delta \sum \ln(h_{it})$ .

Figure 1: Thail entropy index ( $\Delta \ln(\sum h_{it}) - \Delta \sum \ln(h_{it})$ ) and variance of  $\Delta \sum \ln(h_{it})$



If aggregation poses no problem, the term  $\Delta \ln(\sum h_{it}) - \Delta \sum \ln(h_{it})$  should be constant and not correlated with the moments of the distribution or with the instruments. However that is not the case: the autocorrelation coefficient is 0.4 (statistically significant) and it is correlated with the variance of the distribution of  $\Delta \sum \ln(h_{it})$ , as figure 1 shows.

Once this bias is corrected, the estimate is 0.7, closer to the estimates obtained using micro data. At the macro level, Chetty et al (2010) use the studies by Cho and Cooley (1994), King and Rebelo (1999), Smets and Wouters (2007) and Hall (2009) to arrive to a mean estimate of the aggregate Frisch elasticity of 2.61 (which combines labor supply response in both margins). Our results imply a total response of 0.87 (combining both the extensive and the intensive macro Frisch elasticity), which is in line with the evidence presented by Chetty et al (2010) for studies based in microeconomic data.

## 5 Conclusions

We have conducted quasi-empirical experiments using a simulated panel from a stochastic OLG model to estimate the labor supply response to anticipated changes in wages, holding the marginal utility of wealth constant (Frisch elasticity), and to unanticipated change in wages. Using a simulated panel enables us to exactly observe expected wages and control for demand-side variables and uncertainty, as well as to avoid measurement error. The model we used was designed to match the U.S. economy closely, including incomplete market for insurance against earnings shocks, and a Social Security system with payroll taxes and benefits that depend on the actual earnings history. We employed several empirical methods from recent literature.

First, we find that the empirical estimates of Frisch elasticity based on the simulated sample are similar in magnitude to those obtained in empirical studies, and considerably smaller than the theoretical values derived from the utility function used in the model. This result is consistent with other related work, such as Domeij and Floden (2006). We also find that lagged wages are better instruments than life cycle instruments because they contain information about the uncertainty faced by the agents.

Second, we find approximation terms, rather than borrowing constraints, to be the main cause of biases in the Frisch elasticity estimates, a result in contrast to Domeij and Floden (2006). The use of an OLG model with additional insurance mechanisms is key to explain the difference.

Third, the labor supply response to unanticipated wage changes, also important for the analysis of tax and benefit reforms, is very small (0.0007) because of large and age-decreasing wealth effects in the OLG model. In principle, both intertemporal substitution and wealth effects are relevant for determining the labor supply response to tax or benefit changes. A wage increase generates an intertemporal substitution effect which tends to increase the labor supply, but the same wage increase generates a wealth effect and a labor supply reduction. Because reforms often differ at the extent to which they alter these two effects, it is important to explicitly consider measures of wage risk

in the analysis.

Finally, we calculate the macro Frisch elasticity at the extensive margin (work or not) and at the intensive margin (number of hours) using cross sectional aggregation of the participation decisions, hours and the wage rate. For the extensive margin, we obtain an estimate of 0.17, which is close to the microeconomic evidence on extensive margin responses but well below the elasticity implied by other macroeconomic studies. For the intensive margin, we obtain a value of 1.4, and show that this result is due to bias induced the cross sectional aggregation. Once this bias is corrected, the estimate drops to 0.7, half the original value and also consistent with the empirical evidence based in microeconomic studies. Adding both margins, the macro Frisch elasticity is 0.87.

## References

- [1] Altonji, Joseph G. (1986) “Intertemporal substitution in labor supply: Evidence from micro data,” *Journal of Political Economy*, vol. 94, June, S176-S215.
- [2] Angrist, Joshua. (1991) “Grouped data estimation and testing in simple labor supply models,” *Journal of Econometrics*, vol. 47, March.
- [3] Aiyagari, S. Rao. (1994) “Uninsured Idiosyncratic Risk and Aggregate Saving,” *The Quarterly Journal of Economics*, vol. 109(3), August, 659-84
- [4] Attanasio, Orazio and Guglielmo Weber (1993) “Consumption Growth, the Interest Rate and Aggregation,” *Review of Economic Studies*, vol. 60(3), pages 631-49, July.
- [5] Blundell, Richard, Pierre-Andre Chiappori, Thierry Magnac and Costas Meghir. (2007) “Collective labour supply: Heterogeneity and non-Participation,” *Review of Economic Studies*, vol. 74(2), 417-445.
- [6] Blundell, Richard, and Thomas MaCurdy. (1999) “Labor supply: A review of alternative approaches.” In *Handbook of Labor Economics 3*, Amsterdam: Elsevier North Holland.
- [7] Blundell, Richard, and Thomas MaCurdy.(2008) “Labor supply.” In *The New Palgrave Dictionary of Economics*, Palgrave MacMillian.
- [8] Blundell, Richard, Costas Meghir, and Pedro Neves. (1993) “Labor supply: An intertemporal substitution,” *Journal of Econometrics*, vol. 59, 137-160.
- [9] Card, David. (1994) “Intertemporal labor supply: An assessment.” In Christopher Sims, editor, *Advances in Econometrics*, Sixth World Congress. New York: Cambridge University Press.
- [10] Chang, Yongsung, and Sun-Bin Kim. (2006) “From individual to aggregate labor supply: A quantitative analysis based on a heterogeneous agent macroeconomy,” *International Economic Review*, vol. 47, 1-27.

- [11] Chetty, Raj, Adam Guren, Dayanand S. Manoli and Andrea Weber (2011). "Does Indivisible Labor Explain the Difference Between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities," NBER Working Papers 16729
- [12] Chiappori, Pierre-Andre. (1988) "Rational household labor supply," *Econometrica*, vol. 56(1), January, 63-90.
- [13] Chiappori, Pierre-Andre. (1992) "Collective labor supply and welfare," *Journal of Political Economy*, vol. 100(3), June, 437-67.
- [14] Congressional Budget Office. (2008) "Analysis of the president's budgetary proposals for fiscal year 2009."
- [15] Domeij, David, and Martin Floden. (2006) "The labor supply elasticity and borrowing constraints: Why estimates are biased," *Review of Economic Dynamics*, vol. 9, April, 242-262.
- [16] Fehr, Ernst, and Lorenz Götte. (2007) "Do workers work more if wages are high? Evidence from a randomized field experiment," *American Economic Review*, vol. 97, March, 298-317.
- [17] French, Eric. (2004) "The labor supply response to (mismeasured but) predictable wage changes", *Review of Economics and Statistics*, vol. 86, May, 602-613.
- [18] Gourinchas, Pierre-Olivier, and Jonathan Parker. (2002) "Consumption over the life cycle," *Econometrica*, vol. 70, January, 47-89.
- [19] Heckman, James, and Thomas MaCurdy. (1982) "A life cycle model of female labor supply," *Review of Economic Studies*, vol. 49, January, 659-660.
- [20] Heckman, James, and Thomas MaCurdy. (1980) "Corrigendum on a life cycle model of female labor supply," *Review of Economic Studies*, vol. 47, January, 47-74.
- [21] Heckman, James. (1979) "Sample selection bias as a specification error," *Econometrica*, vol. 47, 153-161.

- [22] Heckman, James. (1993) "What has been learned about labor supply in the past twenty years," *American Economic Review (Papers and Proceedings)*, vol. 83, 116-121.
- [23] Huggett, Mark. (1996) "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, vol. 38(3), December, 469-494.
- [24] Imai, Susumu and Michael Keane. (2004) "Intertemporal Labor Supply and Human Capital Accumulation," *International Economic Review*, vol. 45, 601-641.
- [25] Keane, Michael. (2006) "Structural vs. Atheoretical Approaches To Econometrics" forthcoming, *Journal of Econometrics*
- [26] Laitner, John, and Dan Silverman. (2005). "Estimating life cycle parameters from consumption behavior at retirement," Working Papers 099, University of Michigan, Michigan Retirement Research Center.
- [27] Lee, Chul-In. (2001) "Finite sample bias in IV estimation of intertemporal labor supply models: Is the intertemporal substitution elasticity really small?" *The Review of Economics and Statistics*, vol. 83, November, 638-646.
- [28] MaCurdy, Thomas E. (1981) "An empirical model of labor supply in a life cycle setting," *Journal of Political Economy*, vol. 77, December, 1059-1085.
- [29] Nishiyama, Shinichi. (2004) "Analyzing an Aging Population A Dynamic General Equilibrium Approach," CBO Working Paper 2004-3
- [30] Nishiyama, Shinichi, and Kent Smetters, (2005) "Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks," *Journal of Political Economy*, vol. 113(5), October, 1088-1115.
- [31] Pencavel, John. (1986) "Labor supply of men," In Orley Ashenfelter and R. Layard, editors, *Handbook of Labor Economics*, Amsterdam: Elsevier North Holland.
- [32] Pistaferri, Luigi. (2003) "Anticipated and unanticipated wage changes, wage risk, and intertemporal labor supply," *Journal of Labor Economics*, vol. 21, July, 729-728.

- [33] Prescott, Edward. (2006) “Nobel lecture: The transformation of macroeconomic policy and research,” *Journal of Political Economy*, vol. 114, April, 203-235
- [34] Rogerson, Richard, and Johanna Wallenius (2007). “Micro and macro elasticities in a life cycle model with taxes,” NBER Working Paper 13017.
- [35] Ziliak, James, and Thomas Kniesner. (2005) “The effect of income taxation on consumption and labor supply,” *Journal of Labor Economics*, vol. 23, October, 769-796.